ARTICLE NO. 78-1401R

G80-003

10004

Nutation Dampers vs Precession Dampers for Asymmetric Spacecraft

J.E. Cochran Jr.* and J.A. Thompson†
Auburn University, Auburn, Alabama

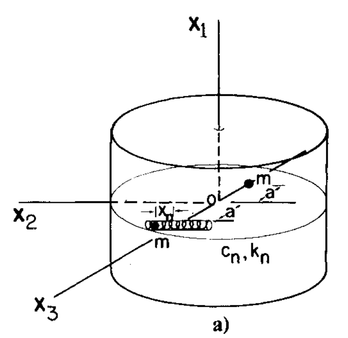
The energy dissipation efficiencies of ball-in-tube dampers oriented with their sensitive axes orthogonal to (nutation) and parallel to (precession) the nominal spin axis of an asymmetric spacecraft are studied using the energy-sink method and numerical integration. Energy-sink equations for each of the two types of dampers are obtained in the forms of damped, periodically forced Hill-type equations. The stability of solutions to Mathieu equation approximations to these equations is considered. An approximate analytical expression for the ratio of the average energy dissipation rates of the two types of dampers is obtained. Results for ideally tuned, identical (except for their spring constants and orientations) dampers are presented and compared with those of a previous study. For a particular spacecraft configuration, analytical results for the energy dissipation ratio are compared with numerical results, and excellent agreement is achieved.

Introduction

IN the design of spin-stabilized spacecraft, an engineer is faced with many problems. One of these is deciding how undesirable attitude motions of the spacecraft, which result when there is a misalignment of the rotational angular momentum of the spacecraft and its angular velocity, are to be removed. This problem is usually solved by supplying some sort of passive device which dissipates energy. The theoretical basis of the use of such devices is grounded in the fact that if the model of quasi-rigid spacecraft is adopted, the minimum rotational kinetic energy state of the spacecraft corresponds to pure spin at a constant rate about its axis of maximum principal moment of inertia. Thus, spin-stabilized spacecraft (which are not of the dual-spin type) are designed to nominally spin about their respective axes of maximum moment of inertia.

The existing dissipation devices are of various types.[1-3] However, the one most often subjected to analysis, and that considered here, is the ball-in-tube damper.[4] Assuming that such a type of device is chosen for use on a particular spacecraft, many other questions such as how should the mechanism itself be designed and how should it be oriented in the spacecraft, must be answered. In this paper, we consider the latter question. In particular, we deal here with the issue of whether the sensitive axis of a ball-in-tube damper, contained in an asymmetric spacecraft, should be oriented parallel or orthogonal to the nominal spin axis of the spacecraft.

Schneider and Likins[5] confronted this issue and used the energy-sink method[6] and numerical integration of equations of motion to obtain information upon which a choice of damper orientation may be based. In doing this they noted that a freely rotating asymmetric rigid body, which is not spinning about one of its principal axes, precesses (here, the coning motion of the principal axis of maximum moment of inertia about the angular momentum) and nutates (here, the *periodic oscillation* in the coning angle). They adopted a spacecraft model which consists of an asymmetric rigid body, a point mass (representing the mass of a ball-in-tube damper), a linear spring, and a linear, viscous damper. The damper mass's degree of freedom was considered to correspond to motion parallel to (precession damper), or orthogonal to (nutation damper), the nominal spin axis of the spacecraft. The energy-sink method was applied, using this model and a "generating solution" in the form of the classical solution for the angular velocity components of a torque-free, asymmetric, rigid body. This solution is in terms of Jacobian elliptic functions of the time. In Ref. 5, these functions were approximated, where they appear in the equations of motion for each of the two types of dampers, by "average" values and trigonometric functions to the extent that two second-order, nonhomogeneous, constant-coefficient, linear, ordinary differential equations were obtained for the dampers.



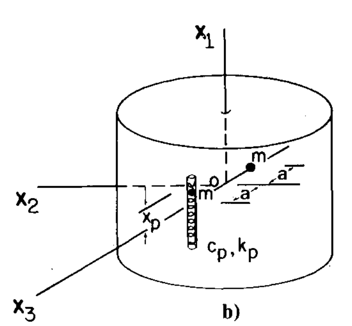


Fig. 1 Damper orientations: a) nutation damper, b) precession damper.

Presented as Paper 78-1401 at the AIAA/AAS Astrodynamics Conference, Palo Alto, Calif., Aug. 7-9, 1978; submitted Nov. 13, 1978; revision received April 20, 1979.

Index category: Spacecraft Dynamics and Control.

*Alumni Associate Professor, Aerospace Engineering. Associate Fellow AIAA.

†Presently, Engineer, McDonnell Douglas Astronautics Co.

The forcing terms of these equations are periodic, with the period of the precession damper forcing function being equal to twice that of the nutation forcing function. An approximate analytical expression for the ratio of the energy dissipation rates of the dampers is given in Ref. 5, and variations of this ratio with an energy parameter for various inertia configurations are presented there. Limited numerical results for the ratio of the dissipation rates are also compared with corresponding analytical results. For some cases, the analytical and numerical results of Ref. 5 are in good agreement; however, for a case in which the coning angle is large and the spacecraft decidedly asymmetric, poor agreement is evident. This was thought by the authors of Ref. 5 to be due to the inaccuracy of the analytical results in that particular case.

In this paper, results of a recent investigation of the same problem treated in Ref. 5 are presented with the objectives of 1) clarifying the definitions of precession and nutation, 2) addressing briefly the question of the stability of the motions of the damper masses, 3) providing a new approximate analytical expression for the ratio of the energy dissipation rates of the two types of dampers which is superior to that of Ref. 5 for large coning angles (and reduces to that of Ref. 5 for small coning angles), and 4) explaining the reason for the poor agreement of numerical and analytical results noted above.

Spacecraft Model

The model adopted for this study differs from that of Ref. 5 in that a second "balance" point mass m is added, and only motion of one damper at a time is considered. This balance point mass is identical to the damper mass but is fixed in the spacecraft. When the system is in its undeformed state, the damper mass and balance mass lie on the axis of least principal moment of inertia of the rigid body, with each a distance a from the center of mass of the rigid body (see Fig. 1). The centroidal principal moments of inertia of the rigid body about the x_1, x_2, and x_3 axes are I_1, I_2, and I_3 ($I_1 > I_2 > I_3$), respectively. The nutation damper is illustrated in Fig. 1a and the precession damper in Fig. 1b. The spring constants (k's) and damping coefficients (c's) of the two types of dampers are distinguished by using the subscripts n (nutation) and p (precession). The displacements of the dampers from their equilibrium positions are denoted by x_n and x_p.

The exact equations of motion for each system are given in the Appendix. They are identical to the corresponding equations [Eqs. (1-5)] of Ref. 5 when those equations are modified by putting $x_n \equiv 0$ (precession damper case) and $x_p \equiv 0$ (nutation damper case), respectively.

Precession and Nutation

In the classical literature (see, for example, Whittaker,[7] page 151) the *precession* of a torque-free, asymmetric, rigid body is the constant part of the time rate of change of the angle ψ, shown in Fig. 2, where C is the mass center of the body, x, y, and z are its centroidal principal axes, and the nonrotating coordinate system $CXYZ$ has its X axis aligned with the rotational angular momentum H of the body. The term "nutation" is classically used, it appears, primarily when an external torque is present as, for example, in the case of a spinning top. However, since nutation is used to refer to "nodding" motion, and since the spin axis (the principal axis of extreme moment of inertia nearest H) of an asymmetric body "nods" in the general case of motion, it is quite common nowadays to refer to the periodic oscillations in the angle θ as nutation. Lemanis[8] does so. The third Eulerian angle Φ is referred to as the "proper rotation" of the Poinsot motion by Lemanis, who gives approximate formulae for the mean values of $\dot{\psi}$, $\dot{\theta}$, and $\dot{\Phi}$.

In applying the energy-sink method to this problem, it turns out that the angular velocity components about the principal axes of a torque-free, asymmetric, rigid body are needed as functions of time, but explicit expressions for the angles ψ, θ, and Φ are not. It should be instructive, therefore, to recall how the angular velocity components, say ω_x, ω_y, and ω_z, are related to the Eulerian angles and their time derivatives. The familiar expressions for the angles chosen are

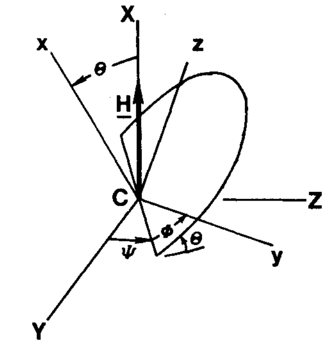


Fig. 2 Eulerian angles.

$$\omega_x = \dot{\Phi} + \dot{\psi}\cos\theta \tag{1a}$$

$$\omega_y = \dot{\theta}\cos\Phi + \dot{\psi}\sin\theta\sin\Phi \tag{1b}$$

and

$$\omega_z = -\dot{\theta}\sin\Phi + \dot{\psi}\sin\Phi\cos\theta \tag{1c}$$

If the principal moments of inertia of the free rigid body are A (about x), B (about y), and C, with $A > B > C$, and if the maximum value of θ is less than $\pi/2$, $\dot{\Phi}$ is negative. Also, ω_x is constant if $B = C$ and oscillates about a mean value when $B \neq C$.

From Ref. 5 and Eqs. (A4) and (A8) (see Appendix), it can be seen that the forcing terms for the *energy-sink equations* (equations of motion for the damper masses with the angular velocity components replaced by their rigid-body approximations) are proportional to $\dot{\omega}_x - \omega_y\omega_z$ (nutation) and $\dot{\omega}_y + \omega_x\omega_z$ (precession). Since, from Euler's equations for a free rigid body, $\dot{\omega}_x = [(B-C)/A]\omega_y\omega_z$ and $\dot{\omega}_y = [(C-A)/B]\omega_x\omega_z$, they are thus proportional to $\omega_y\omega_z$ (nutation) and $\omega_x\omega_z$ (precession).

In terms of the Eulerian angles and their time derivatives, the ratio of the two forcing terms (nutation/precession) is proportional to

$$\omega_y/\omega_x = (\dot{\theta}\cos\Phi + \dot{\psi}\sin\theta\sin\Phi)/(\dot{\Phi} + \dot{\psi}\cos\theta)$$

For small values of θ, $\dot{\theta}$ is also small and $\dot{\Phi}$ and $\dot{\psi}$ almost constant. The *amplitude* of the ratio is thus approximately proportional to θ. For larger values of θ, even for axisymmetric bodies (for which θ is constant), it appears that the amplitude of the ratio *may* be greater than unity. It is rather easy to show that, for axisymmetric bodies, the ratio of the amplitudes‡ of the forcing terms (Eqs. (5) which follow) is $[4C/A\text{-}2]^{-1}\tan\theta$.

Energy-Sink Results

The equations for the ω_j, $j = 1,2,3$, given in the Appendix, can be put into the forms of Eqs. (2). We note that, although there are terms in the rotational motion equations which *appear* to be zeroth order in m, e.g., $[a/(1-\mu)]$ $[c_n\dot{x}_n + k_n x_n]$, by considering the equations which govern the translation of the damper masses, it is evident that these also are first order in m.

$$\dot{\omega}_1 = \frac{(B-C)}{A}\omega_2\omega_3 + \epsilon\frac{C}{A}f_1 + O(\epsilon^2) \tag{2a}$$

‡The amplitudes of the forcing terms are $[ma(H^2/C^2)\sin^2\theta]/2$ and $ma\,H^2/(AC)\sin\theta\cos\theta$, when $B = C$.

$$\dot{\omega}_2 = \frac{(C-A)}{B}\omega_3\omega_1 + \epsilon\frac{C}{B}f_2 + O(\epsilon^2) \quad (2b)$$

and

$$\dot{\omega}_3 = \frac{(A-B)}{C}\omega_1\omega_2 + \epsilon f_3 + O(\epsilon^2) \quad (2c)$$

In Eqs. (2), $A=I_1$, $B=I_2$, $C=I_3$, $\epsilon = a^2m/C$. Also, the f_j are functions of the ω_k and either x_n and $\dot{x}_n$, or x_p and $\dot{x}_p$, depending on the type of damper. The f_j are O(1). From Eqs. (2), it is evident that the motion of the spacecraft can be considered to be that of a rigid body with moments of inertia A, B, and C, $A>B\geq C$, perturbed by disturbing angular accelerations which are small if ϵ is small. It may be expected, therefore, that the motion of the spacecraft is very nearly free-Eulerian for small periods of time. The energy-sink method may be considered to be based on this observation.

More exactly, the energy-sink method is predicated on the assumption that the energy and angular momentum due to rotational motion only are varying slowly enough, that the zeroth-order solution for the ω_j may be used in the damper-mass translational equations to obtain $\dot{x}_n$ and $\dot{x}_p$ approximately as functions of time. Since the rate at which energy is dissipated by the nutation (precession) damper is ½ $c_n\dot{x}_n^2$ (½ $c_p\dot{x}_p^2$), the average value of this is taken as a measure of the rate at which energy is dissipated.

Zeroth-Order Solution for the ω_j

The zeroth-order solutions for the ω_j, j = 1,2,3, are denoted by ω_x, ω_y, ω_z, respectively, where the latter comprise the solution to Euler's equations for a torque-free, asymmetric, rigid body. This solution may be found in many references on classical mechanics. The source chosen for this study is Ref. 9 because it was used by the authors of Ref. 5. The solution is

$$\omega_x = \alpha \, \text{dn}\, u \quad (3a)$$

$$\omega_y = \beta \, \text{sn}\, u \quad (3b)$$

$$\omega_z = -\gamma \, \text{cn}\, u \quad (3c)$$

where

$$\alpha = \sqrt{(H^2-2CT)/[A(A-C)]}$$

$$\beta = \sqrt{(2AT-H^2)/[B(A-B)]}$$

$$\gamma = \sqrt{(2AT-H^2)/[(C(A-C)]}$$

$$u = p(t-t_0) \qquad p = \sqrt{(H^2-2CT)/(ABC)}$$

and dn u, sn u, and cn u are Jacobian elliptic functions of modulus,

$$k = \sqrt{[(2AT-H^2)(B-C)]/[(H^2-2CT)(A-B)]}$$

Here, T is the rotational kinetic energy; i.e., $T = 1/2(A\omega_x^2 + B\omega_y^2 + C\omega_z^2)$, and H is the magnitude of the rigid-body angular momentum; i.e., $H^2 = A^2\omega_x^2 + B^2\omega_y^2 + C^2\omega_z^2$. Also, t_0 is the time when $\omega_y = 0$. We note that this solution corresponds to motions "about the axis of maximum moment of inertia," in which case $H^2 > 2BT$.

Since $\omega_x = (H/A)\cos\theta$, $\omega_y = (H/B)$ $\sin\theta$ $\sin\Phi$, and $\omega_z = (H/C)$ $\sin\theta\cos\Phi$, Eqs. (3) define the coning angle θ and the angle of spin, or "proper rotation," Φ. The precession angle ψ is *not involved in* ω_x, ω_y, or ω_z explicitly. However, $\dot{\psi}$ is the major contributor of the Eulerian angle rates to H. In fact, $H = \dot{\psi}BC/(C\cos^2\Phi + B\sin^2\Phi)$ (Ref. 10).

Energy-Sink Equations for Dampers

When the energy-sink method is applied to this problem, the "forcing terms" which appear in the linear, ordinary differential equations with variable coefficients that are obtained for the damper masses may be expressed as

$$f_n = ma(\dot{\omega}_x - \omega_y\omega_z) \qquad \text{(nutation)} \quad (4a)$$

and

$$f_p = -ma(\dot{\omega}_y + \omega_x\omega_z) \qquad \text{(precession)} \quad (4b)$$

respectively. Also, elsewhere in the exact damper equations the ω_j are replaced by their zeroth-order solutions.

Since ω_x, ω_y, and ω_z satisfy Euler's equations, we may rewrite f_n and f_p in either the forms

$$f_n = -ma(1 + C/A - B/A)\omega_y\omega_z \quad (5a)$$

and

$$f_p = -ma(1 + C/B - A/B)\omega_x\omega_z \quad (5b)$$

or the forms

$$f_n = -ma[(A+C-B)/(B-C)]\dot{\omega}_x \quad (6a)$$

and

$$f_p = -ma[(B+C-A)/(C-A)]\dot{\omega}_y \quad (6b)$$

The reason for writing f_n and f_p in the latter forms will become obvious shortly.

Fourier series for dn u and sn u are given in the Appendix. (As indicated there, these and other results involving elliptic functions are taken from Ref. 11.) Also given in the Appendix is an expression for $\text{dn}^2\, u$ in terms of the complete elliptic integrals of the first and second kinds (K and E, respectively) of modulus k and the derivative of Jacobi's zeta function $Z(u)$, which is periodic in u with period $2K$. The facts that dn u and sn u are periodic in u with periods $2K$ and $4K$, respectively, are evident in the series expansions of these functions. Jacobi's nome, $q = e^{-\pi K'/K}$, where K' is the complete elliptic integral of the first kind of modulus $k' = \sqrt{1-k^2}$, appears in the series and, due to its smallness for even relatively large values of k, the convergence of each of the series is generally very rapid.

Because the functions dn u, sn u, and $Z(u)$ are continuous with continuous derivatives, and since $\text{dn}(K) = \text{dn}(-K)$, $\text{sn}(2K) = \text{sn}(-2K)$, and $Z(K) = Z(-K)$, their derivatives with respect to u are equal to the derivatives term by term of their respective Fourier series.[12] Hence,

$$\dot{\omega}_x = \frac{2\alpha\pi^2}{K^2} pq \sum_{j=0}^{\infty} \frac{q^j 2(j+1)}{1+q^{2(j+1)}} \sin\frac{2(j+1)\pi u}{K} \quad (7a)$$

and

$$\dot{\omega}_y = \frac{\beta p \pi^2}{kK^2} q^{1/2} \sum_{j=0}^{\infty} \frac{q^j(2j+1)}{1-q^{2j+1}} \cos\frac{(2j+1)\pi u}{2K} \quad (7b)$$

Note that the period, in time, of $\dot{\omega}_y$ is $4K/p$ and that of $\dot{\omega}_x$ is $2K/p$. Also, for small k, $q \simeq k^2/16$ and $\dot{\omega}_y$ reduces to $\beta p\cos u$ as k approaches zero.

Turning now to the zeroth-order damper mass equations, we first write them in the forms ($\mu \ll 1$ is neglected)

$$m\ddot{x}_n + c_n\dot{x}_n + [\lambda_n - qg_n(t)]x_n = f_n(t) \quad (8a)$$

and

$$m\ddot{x}_p + c_p\dot{x}_p + [\lambda_p + q\, g_p(t)]x_p = f_p(t) \quad (8b)$$

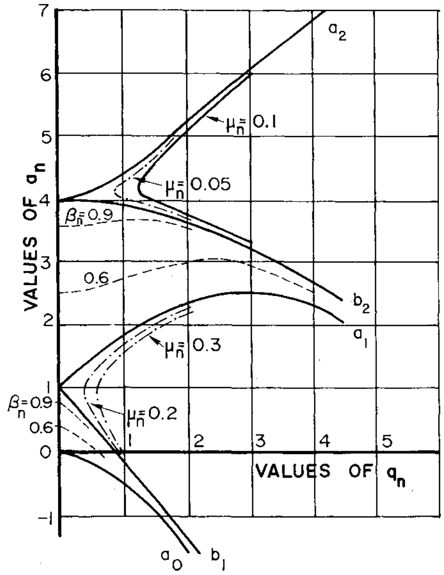


Fig. 3 Stability diagram.

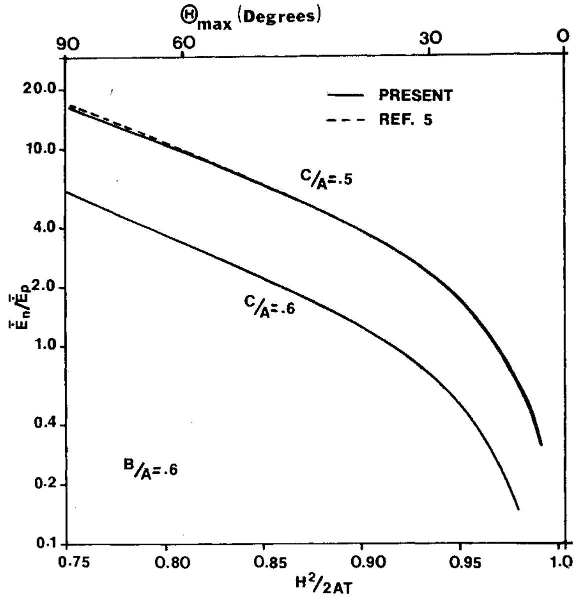


Fig. 4 Comparison of previous and present analytical results for $\bar{E}_n/\bar{E}_p$.

where

$$\lambda_n = k_n - m[(\alpha^2 + \gamma^2/k^2)E/K - \gamma^2 k'^2/k^2]$$

$$\lambda_p = k_p - m\{(\beta^2 - \gamma^2 k'^2)/k^2 - [(\beta^2 - \gamma^2)/k^2]E/K\}$$

$$g_n = \frac{2\pi^2(\alpha^2 + \gamma^2/k^2)}{K^2} \sum_{j=1}^{\infty} m\left(\frac{jq^{j-1}}{1-q^2 j}\right)\cos\frac{j\pi u}{K}$$

$$g_p = \frac{2(\beta^2 - \gamma^2)\pi^2}{k^2 K^2} \sum_{j=1}^{\infty} m\left(\frac{jq^{j-1}}{1-q^2 j}\right)\cos\frac{j\pi u}{K}$$

and $f_n(t)$ and $f_p(t)$ are given by Eqs. (6) with $\dot{\omega}_x$ and $\dot{\omega}_y$ replaced by Eqs. (7). As k approaches zero due to a decrease in the coning angle, both g_n and g_p remain finite, and q approaches zero. However, as k approaches zero due to decreasing asymmetry, since g_n is of order $1/(B-C)$ and q is of order $B-C$, the product qg_n reduces to the axisymmetric value, $m[(\gamma^2/2)\cos 2p(t-t_0)]_{B=C}$.

Approximate Energy-Sink Equations with Variable Coefficients

Equations (8) are periodically forced Hill-type equations,[13] the solutions of which are beyond the scope of this paper. For q sufficiently small, they may be approximated by Mathieu equations. Since a relatively great deal is known about Mathieu equations (in particular, information concerning the stability of their solutions), we next consider these approximations. Cloutier[14] has studied the stability of solutions to similar equations for axisymmetric spacecraft.

We let $z = \pi u/2K$, $(\ )' = d(\ )/dz$, $y_n = e^{\sigma_n z} x_n$, where $\sigma_n = c_n K/(m\pi p)$, and neglect terms of higher order than the first in q which appear in Eqs. (8) to obtain the approximate equation

$$y_n'' + (a_n - 2q_n \cos 2z) y_n = e^{\sigma_n z} A_n \sin 2z \tag{9a}$$

In Eq. (9a),

$$a_n = 4\lambda_n K^2/(m\pi^2 p^2) - \sigma_n^2$$

$$q_n = 4q[C/(A-B) + A/(B-C)]B/(A-C)$$

and

$$A_n = 8qa(\gamma\beta - \alpha p k^2)/(k^2 p^2)$$

If the nutation damper is tuned to the nutation frequency $\pi p/K$, then $a_n = 4 - \sigma_n^2$.

The general solution to Eq. (9a) when $A_n = 0$ may be expressed in the form

$$y_n = C_1 e^{\mu_n z}\phi(z) + C_2 e^{-\mu_n z}\phi(-z)$$

where C_1 and C_2 are arbitrary constants, $\mu_n = \mu_n(a_n, q_n)$ is a real constant, and $\phi(z)$ is a periodic function, or in the form obtained by replacing μ_n by $i\beta_n$, where β_n is a real constant and $i = \sqrt{-1}$. Since $x_n = e^{-\sigma_n z} y_n$, the approximate solution for x_n derived from Eq. (9a) will be stable if σ_n is greater than the magnitude of μ_n. Figure 3 is a reproduction of a portion of Fig. 10 of Ref. 13 with McLachlan's notation altered to agree with ours. The regions containing μ_n curves are "unstable" regions. The diagram is symmetrical with respect to the a_n axis. Note that if $\sigma_n = 0$, for a tuned damper, instability exists when $q_n \neq 0$. For $\sigma_n > 0$, the solution for x_n is stable if the point (a_n, q_n) lies in the region which contains the curves labeled β_n, or if it lies to the *left* of μ_n curves for which $|\mu_n| < \sigma_n$.

An equation similar to Eq. (9a) can be obtained from Eq. (8b). This equation is

$$y_p'' + (a_p - 2q_p \cos 2z) y_p = e^{\sigma_p z} A_p \cos z \tag{9b}$$

where (for a tuned damper)

$$a_p = 1 - \sigma_p^2 \qquad \sigma_p = c_p K/(m\pi p)$$

$$q_p = -4qA(B+C-A)/[(A-C)(A-B)]$$

and

$$A_p = 4q^{1/2} a(\alpha\gamma - p\beta)/(k^2 p^2)$$

A plot identical to that in Fig. 3, except reflected about the ordinate, may be constructed with a_p, q_p, μ_p and β_p as parameters.

Since the unstable region originating at the point (1,0) expands more rapidly with decreasing q_p than that region originating at (4,0), it appears that relatively more damping should be supplied for the precession damper than for the nutation damper. For the precession damper, however, the magnitude of the parameter q_p is small compared to q_n for reasonable coning angles, say $\theta < 30$.

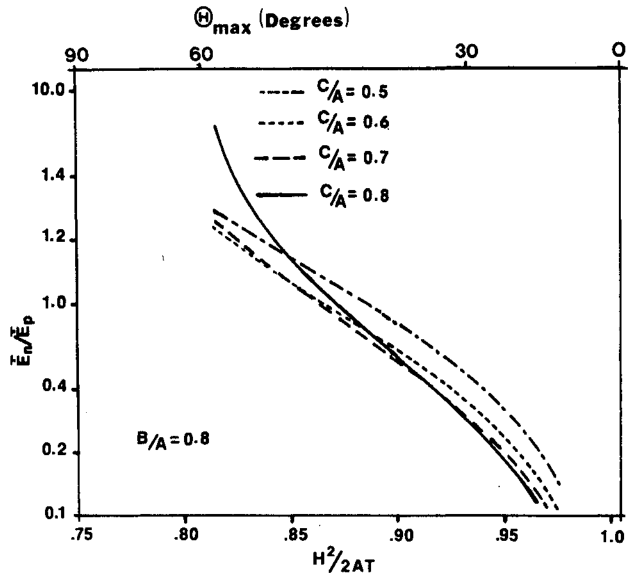


Fig. 5 Family of $\bar{\dot{E}}_n/\bar{\dot{E}}_p$ curves for $B/A=0.8$.

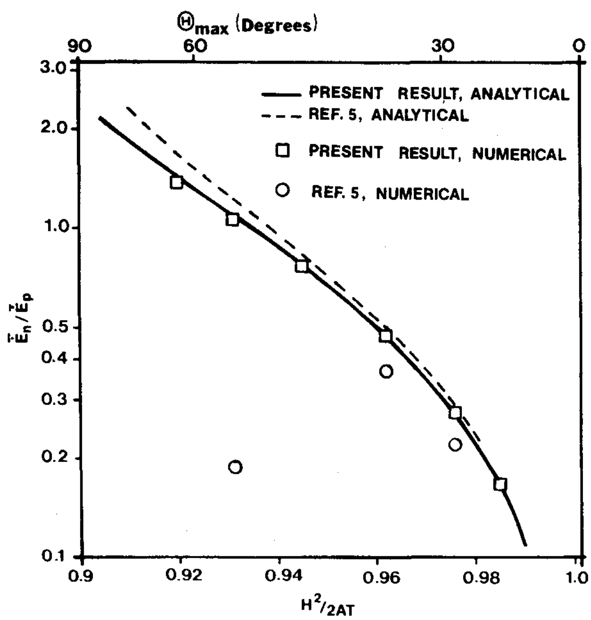


Fig. 6 Comparison of analytical and numerical results.

Constant Coefficient Approximation

Although further study of the Mathieu equations may be warranted, we shall now consider the periodic coefficients to be negligible compared to the λ's. This *is* the case if q is sufficiently small. One reason for doing this is that constant coefficient equations, which are easily solved, result when the periodic coefficients are neglected. A second reason for dropping the periodic coefficients is that such terms were neglected by the authors of Ref. 5, and we wish to compare our results with theirs from the viewpoint of determining the effects of how the elliptic functions are approximated is the ultimate results.

It is emphasized here that in Ref. 5 the elliptic functions were well approximated for small values of k. But, fairly large values of k, say $k>0.5$, may correspond to the coning angle at which nutation damping is superior to precession damping. Therefore, a more accurate approximation is desirable.

As alluded to previously, "tuning" of the dampers should *ideally* be accomplished by choosing k_n and k_p so that $\sqrt{\lambda_n/m}$ and $\sqrt{\lambda_p/m}$ are, respectively, equal to the frequencies $\pi p/K$ and $\pi p/2K$. In Ref. 5, they were assumed tuned to the frequencies $2p$ and p, respectively. Practically speaking, the dampers will have to be tuned to specific frequencies, unless a means of changing the spring constants can be provided. If, for example, $\sqrt{k_n/m}$ is chosen when $\theta=0$; i.e., $2\omega_0\sqrt{(A-C)/BC}$, where ω_0 is the spin rate, then, for sufficiently large values of θ, λ_n will be negative. In that case, unless the damping is sufficient to produce $\sigma_n>|\mu_n|$, an unstable motion of the damper mass of one of the types investigated by Cloutier[14] will occur.

If Eqs. (8) are approximated by constant-coefficient equations, but their *entire right-hand sides are retained,* by a straightforward process approximate steady-state solutions for x_n and x_p can be obtained. These can then be used to obtain steady-state approximations to $\dot{x}_n$ and $\dot{x}_p$. Finally, by averaging the squares of these approximate rates, multiplied by the appropriate damping coefficients, over the periods of $2K/p$ and $4K/p$, respectively (and after a considerable amount of algebra), the following expressions for the ratio of the average energy dissipation rates $\bar{\dot{E}}_n$ and $\bar{\dot{E}}_p$ is found:

$$\bar{\dot{E}}_n/\bar{\dot{E}}_p=16\frac{c_n}{c_p}\left[\frac{A+B-C}{B+C-A}\right]^2\left[\frac{B(C-A)}{A(B-C)}\right]\frac{qb(q)}{c(q)} \qquad (10)$$

where

$$b(q)=\sum_{j=0}^{\infty}\frac{[q^j(j+1)/(1+q^{2(j+1)})]^2}{\{[\lambda_n-m(j+1)^2(\pi p/K)^2]^2+[c_n(j+1)\pi p/K]^2\}}$$

and

$$c(q)=\sum_{j=0}^{\infty}\frac{[q^j(2j+1)/(1-q^{2j+1})]^2}{\{[\lambda_p-m(2j+1)^2(\pi p/2K)^2]^2+[c_p(2j+1)\pi p/2K]^2\}}$$

For k small enough that terms of order k^4 may be neglected compared to k^2, the ratio given by Eq. (10) is identical to that of Ref. 5. Also, for the asymmetric case $B=C$, we get from Eq. (10) the result,

$$(\bar{\dot{E}}_n/\bar{\dot{E}}_p)_{B=C}=\tfrac{1}{4}(c_p/c_n)\{\tan^2\theta/[2C/A-1]^2\} \qquad (11)$$

Furthermore, for q small enough that terms of order higher than the first in it are negligible,

$$\bar{\dot{E}}_n/\bar{\dot{E}}_p\simeq 4\frac{c_p}{c_n}\left[\frac{A+B-C}{B+C-A}\right]^2\left[\frac{B(A-C)}{A(B-C)}\right]q \qquad (12)$$

Results for Various Inertia Combinations when $c_p=c_n$

The moments of inertia are subject to the physical constraint $B/A+C/A>1$ and the *imposed* constraint $1>B/A>C/A$. We recall also that $H^2/2BT>1$, for the motion considered. Hence, $H^2/2AT>B/A$. Also, the maximum coning angle achievable for a given initial state is

$$\theta_{\max}=\cos^{-1}\left[\frac{H^2/2AT-B/A}{(1-B/A)H^2/2AT}\right]^{1/2}$$

The ratios given by Eq. (10) (only five terms in the series) as functions of $H^2/2AT$ for $B/A=0.6$ and $C/A=0.5$ and 0.6 are shown in Fig. 4 for the case $c_p=c_n$. Also shown are some results taken from Ref. 5. There is very little difference in corresponding curves for $C/A=0.5$ (Ref. 5 and ours) when $H^2/2AT$ is very near unity. Only one curve for $C/A=0.6$ is shown because, for axisymmetric spacecraft $k=0$ and the two approximate expressions both reduce to Eq. (11).

The curve for $B/A=C/A=0.6$ in Fig. 4 illustrates the fact that even for axisymmetric spacecraft the nutation damper is superior when the coning angle is sufficiently large. From Eq. (11), with $c_p = c_n$, the "critical" angle θ_c above which nutation damping becomes superior is

$$\theta_c = \tan^{-1}\{4C/A - 2\} \tag{13}$$

For $C/A = 0.6$, $\theta_c \simeq 21.80$ deg.

A family of energy dissipation ratio curves for $B/A = 0.8$ is presented in Fig. 5. The more asymmetric the spacecraft is, i.e., the larger $B/A - C/A$, the more the corresponding curve is shifted upward. This characteristic may also be seen from Eq. (12), since for smaller C/A, with B/A fixed, the squared factor is larger. The $(B-C)$ factor in the denominator of the next term is cancelled by a like factor in q. We note, however, that when the spacecraft is nearly axisymmetric (say, $C/A = 0.7$) the curve crosses those for lower C/A values. For an axisymmetric spacecraft, the ratio curve crosses the other curves and $\bar{\dot{E}}_n/\bar{\dot{E}}_p$ becomes infinite at $H^2/2AT = C/A$.§ This is to be expected, because when $B = C$ the coning angle is 90 deg when $H^2/2AT = C/A$. Hence, the spacecraft is spinning about its y-axis (or z-axis) and the dampers have exchanged roles. Because the smaller coning angles are of more interest, the shifting of the curves due to asymmetry is an important characteristic.

Comparison of Analytical and Numerical Results

To study the accuracy of the analytical expressions for $\bar{\dot{E}}_n/\bar{\dot{E}}_p$, the case $C/A = .275$, $B/A = 0.9$ was investigated numerically. The exact equations of motion for each of the systems of spacecraft plus damper were integrated numerically, the average values of $\dot{E}_n$ and $\dot{E}_p$ were calculated after steady-state behavior was achieved. A fourth-order, Runge-Kutta algorithm with step-size control[16] was chosen for the numerical integration of the equations of motion, while average values $\bar{\dot{E}}_n$ and $\bar{\dot{E}}_p$ were obtained by simple Euler integration of $\dot{E}_n$ and $\dot{E}_p$ over the characteristic period involved with subsequent division by the interval of integration. The dampers were *ideally* tuned.

In addition to the inertia ratios stated, the system parameters included $A = 1713.8$ kg$-$m^2, $\mu = 0.26 \times 10^{-3}$, $c_n = c_p = 0.7292$ N-s/m, $m = 0.146$ kg, and $a = 1.524$ m. In the process of generating some preliminary results with $c_n = c_p = 0.00729$ and values of $H^2/2AT < 0.92$, unstable behavior, consistent with predicated behavior of the solutions to the approximate Mathieu equations, was found. The larger damping coefficients cited were therefore used to achieve stable motions of the damper masses. The analytical results of Ref. 5 are independent of the value of the damping coefficients if $c_n = c_p$. Equation (10) is not. However, the differences in the ratios $\bar{\dot{E}}_n/\bar{\dot{E}}_p$ obtained using only the first terms in the series in Eq. (10) and using five terms are not distinguishable on a plot the size of that in Fig. 6. Furthermore, the numerical results, for values of $H^2/2AT$ such that unstable behavior is not obtained with the smaller $c_n = c_p$, are likewise indistinguishable from those obtained with the larger value.

Our analytical solution for $\bar{\dot{E}}_n/\bar{\dot{E}}_p$ and that of Ref. 5 are shown in Fig. 6 along with the values obtained numerically in this investigation and the previous one.[5] Excellent agreement between our analytical and numerical results is apparent, even for values of $H^2/2AT$ near the limiting value of 0.90. We also note that there is very good agreement between our results and the analytical results of Ref. 5 for smaller coning angles. The previous numerical results for $H^2/2AT \cong 0.962$, and especially $H^2/2AT \simeq 0.932$, differ significantly from the analytical results. This is due to the detuning of the dampers—for example, if $H^2/2AT = 0.9315$ and $K = 2.021$. Hence, due to only the differences in the frequencies of the forcing functions and those of dampers tuned to $2p$ and p, the detuning is greater than 22%. Some additional detuning is caused by the choice of spring constants on the basis of "average" values of $\omega_1^2 + \omega_3^2$ and $\omega_2^2 + \omega_3^2$ which are not exact.

§The crossing behavior of these curves has been verified numerically using exact equations of motion.[15]

Conclusions

For sufficiently large coning angles, the average rate at which energy is dissipated by an ideally tuned ball-in-tube damper oriented with its sensitive axis orthogonal to the nominal spin axis of either an asymmetric or axisymmetric spacecraft is greater than that of a damper which is identical (except for its spring constant) and oriented with its sensitive axis parallel to the nominal spin axis of the same spacecraft. Asymmetry of the spacecraft decreases the magnitude of the coning angle at which the dampers are equally efficient.

To achieve optimum performance from a damper with either orientation, it must be tuned to a frequency which depends on 1) the inertia characteristics of the spacecraft, 2) the magnitude of the coning angle (i.e., the ratio $H^2/2AT$), and 3) the magnitude of H. If either of the dampers is tuned to a constant frequency and required to operate over a range of coning angles, then, in addition to degradation in performance due to detuning, unstable motions of the damper mass, constrained only by the tube, may occur.

Since there may be instances when large coning angles occur (during reorientation, for example), nutation and precession dampers on the same spacecraft, or a single damper with skewed sensitive axis, could be utilized. The analytical results presented here should prove useful in designing such systems.

Appendix

Equations of Motion

The equations of motion for the system when a nutation damper is utilized are

$$\begin{aligned}&\{I_1 + ma^2[(1-2\mu)/(1-\mu)] + m(1-\mu)x_n^2\}\dot{\omega}_1\\ &-\{I_2 - I_3 + ma^2[(1-2\mu)/(1-\mu)] - m(1-\mu)x_n^2\}\omega_2\omega_3\\ &+[2m(1-\mu)x_n\omega_1 + ac_n/(1-\mu)]\dot{x}_n\\ &+a[k_n/(1-\mu) - m(\omega_1^2+\omega_2^2)]x_n = 0\end{aligned} \tag{A1}$$

$$\begin{aligned}&(I_2+2ma^2)\dot{\omega}_2 - max_n\dot{\omega}_3 - [I_3 - I_1 - 2ma^2]\omega_1\omega_3\\ &+max_n\omega_1\omega_2 - 2ma\dot{x}_n\omega_3 = 0\end{aligned} \tag{A2}$$

$$\begin{aligned}&-max_n\dot{\omega}_2 + [I_3 + m(1-\mu)x_n^2]\dot{\omega}_3 - [I_1 - I_2 + m(1-\mu)x_n^2]\omega_1\omega_2\\ &+2m(1-\mu)x_n\dot{x}_n\omega_3 - max_n\omega_1\omega_3 = 0\end{aligned} \tag{A3}$$

and

$$\begin{aligned}&m(1-\mu)\ddot{x}_n + c_n\dot{x}_n + [k_n - m(1-\mu)(\omega_1^2+\omega_3^2)]x_n\\ &\quad = ma(\dot{\omega}_1 - \omega_2\omega_3)\end{aligned} \tag{A4}$$

where $\mu = m/(M+2m)$, m is the damper mass, and M is the mass of the rigid-body portion of the spacecraft.

The analogous equations for the system when a precession damper is present are

$$\begin{aligned}&[I_1+2ma^2]\dot{\omega}_1 - max_p\dot{\omega}_3 - [I_2 - I_3 + 2ma^2]\omega_2\omega_3\\ &-max_p\omega_1\omega_2 - 2ma\dot{x}_p\omega_3 = 0\end{aligned} \tag{A5}$$

$$\begin{aligned}&\{I_2 + ma^2[(1-2\mu)/(1-\mu)] + m(1-\mu)x_p^2\}\dot{\omega}_2\\ &-\{I_3 - I_1 - ma^2[(1-2\mu)/(1-\mu)] + m(1-\mu)x_p^2\}\omega_1\omega_3\\ &+[2m(1-\mu)x_p\omega_2 - ac_p/(1-\mu)]\dot{x}_p - a[k_n/(1-\mu)\\ &-m(\omega_1^2+\omega_2^2)]x_p = 0\end{aligned} \tag{A6}$$

$$[I_3+m(1-\mu)x_p^2]\dot{\omega}_3-max_p\dot{\omega}_1-[I_1-I_2-m(1-\mu)x_p^2]\omega_1\omega_2$$
$$+2m(1-\mu)x_p\dot{x}_p\omega_3+max_p\omega_2\omega_3=0 \quad \text{(A7)}$$

and

$$m(1-\mu)\ddot{x}_p+c_p\dot{x}_p+[k_p-m(1-\mu)(\omega_2^2+\omega_3^2)]x_p=$$
$$-ma(\dot{\omega}_2+\omega_1\omega_3) \quad \text{(A8)}$$

Fourier Series

The Fourier series used in the body of this paper are those for dn u, sn u, and dn^2 u. The series for dn u and sn u can be taken directly from Ref. 11, pages 304-305, and are

$$\text{sn}\ u=\frac{2\pi}{(kK)}\sum_{j=0}^{\infty}\frac{q^{j+\frac{1}{2}}}{1-q^{2j+1}}\sin\frac{(2j+1)\pi u}{2K} \quad \text{(A9)}$$

and

$$\text{dn}\ u=\frac{\pi}{K}\left[\tfrac{1}{2}+\sum_{j=0}^{\infty}\frac{2q^{j+1}}{1+q^{2(j+1)}}\cos\frac{(j+1)\pi u}{K}\right] \quad \text{(A10)}$$

respectively.

Although cn^2u and sn^2u appear in the energy-sink equations, the identities $\text{cn}^2u+\text{sn}^2u=1$ and $\text{dn}^2u=\sqrt{1-k^2\text{sn}^2u}$ can be used to express these functions in terms of dn^2u. From page 33 of Ref. 11,

$$Z(u)=\int_0^u\left(\text{dn}^2u-\frac{E}{K}\right)\text{d}u \quad \text{(A11)}$$

so that

$$\text{dn}^2u=E/K+\partial Z(u)/\partial u \quad \text{(A12)}$$

where the partial derivative is used to indicate that only u (and not k also) is involved in the differentiation. Now, from page 301 of Ref. 11,

$$Z(u)=\frac{2\pi}{K}\sum_{j=1}^{\infty}\frac{q^j}{1-q^{2j}}\sin\frac{j\pi u}{K} \quad \text{(A13)}$$

Thus,

$$\text{dn}^2u=\frac{E}{K}+\frac{2\pi^2}{K^2}\sum_{j=1}^{\infty}\frac{jq^j}{1-q^{2j}}\cos\frac{j\pi u}{K} \quad \text{(A14)}$$

Equations (A9), (A10), and (A14) are the required series.

Acknowledgments

This research was supported in part by the Auburn University Engineering Experiment Station.

References

[1] Alfriend, K.T., "Partially Filled Viscous Ring Nutation Damper," *Journal of Spacecraft and Rockets,* Vol. 11, July 1974, pp. 465-562.

[2] Sen, A.K., "Stability of a Dual-Spin Satellite with a Four Mass Nutation Damper," *AIAA Journal,* Vol. 8, April 1970, pp. 822-823.

[3] Haines, G.A. and Leondes, C.T., "Eddy Current Nutation Dampers for Dual Spin Satellites," *The Journal of Astronautical Sciences,* Vol. XXI, No. 1, July-Aug. 1973, pp. 1-25.

[4] Cloutier, G.T., "Stable Rotation States of Dual Spin Spacecraft," *Journal of Spacecraft and Rockets,* Vol. 5, April 1968, pp. 490-492.

[5] Schneider, C.C. and Likins, P.W., "Nutation Dampers vs. Precession Dampers for Asymmetric Spacecraft," *Journal of Spacecraft and Rockets,* Vol. 10, March 1973, pp. 218-222.

[6] Iorillo, A.J., "Development of Hughes' Gyrostat System," Hughes Aircraft Company, SSD70012 R, Jan. 1967.

[7] Whittaker, E.T., *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies,* Cambridge University Press, London, 1937.

[8] Lemanis, E., *The General Problem of the Motion of Coupled Rigid Bodies About a Fixed Point,* Springer-Verlag, New York, 1965.

[9] Synge, J.L. and Griffith, B.A., *Principles of Mechanics,* McGraw-Hill, New York, 1959, pp. 377-380.

[10] Cochran, J.E., "Rotational Equations of Motion for a Triaxial Rigid Body," *AIAA Journal,* Vol. 9, June 1971, pp. 1195-1197.

[11] Byrd, P.F. and Friedman, M.D., *Handbook of Elliptic Integrals for Engineers and Scientists,* 2nd Ed. (rev.), Springer-Verlag, New York, 1971.

[12] Bylerly, W.E., *An Elementary Treatise on Fourier's Series,* Dover Publications, New York, 1959, pp. 67-68.

[13] McLachlan, N.W., *Theory and Applications of Mathieu Functions,* Oxford University Press, London, 1947.

[14] Cloutier, G.J., "Nutation Damper Instability on Spin-Stabilized Spacecraft," *AIAA Journal,* Vol. 7, Nov. 1969, pp. 2110-2115.

[15] Thompson, J.A., "Analysis of Nutation and Precession Dampers for Spin-Stabilized Spacecraft," Master's Thesis, Auburn University, Auburn, Alabama, Dec. 1978.

[16] "System/360 Scientific Subroutine Package (360A-CM-03X) Version III Programmers' Manual," International Business Machines Corp., White Plains, New York, 1968.